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Fast & Dynamic Image Restoration using Laplace equation Based Image Inpainting

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ABSTRACT

Images tend to degenerate over time and are exposed to noises. These noises don't only affect the visual outlook but also hampers the allied significance to these images. Different techniques are being put to use for de-noising such images. Image in-painting is one such phenomenon of de-noising the images that involves approximating the de-noised form of the image. The current study aims at developing an in-painting system for restoration of lost art, reconstruction of destroyed images and removal of unnecessary objects. The motivation for the same has been driven from Partial differential equation based anisotropic diffusion model or image in-painting .The steady state heat equation or the Laplace equation has been used to model and approximate the de-noised data for noised region of the image. The Laplace equation has been used clubbed with the Dirichlet boundary conditions in order to fill in the degenerated or the noised region. The Dirichlet conditions provides a firm starting point for approximating the structural framework of the noised region in order to remove the inconsistencies using the colour intensities of the neighboring pixels as the boundary conditions. The colour intensities are modeled to be diffusive in nature. The experiments conducted using the proposed approach portray significant speedups and presents a practicable in-painting strategy.

Keywords**:** De-noising, Laplace Equation, Image Restoration, Inpainting, Partial Differential Equations.

INTRODUCTION

Partial differential equations are often used to model most of the basic theories underlying physics and engineering. We are enabled to simulate various phenomena comprising a whole range of domains viz-a-viz electronics, image processing, etc. We will be dealing with the Laplace equations in this paper as part of the strategy to achieve image inpainting. Inpainting is the digital art of reconstructing old images by filling up the missing portions in a legible and unified manner. The process uses various types of differential equations to delete an unwanted object in the image or fill up a break in the picture. Also referred to as image interpolation, inpainting uses sophisticated algorithms to correct or replace the lost or corrupted data in an image. The process has huge importance in restoring lost artefacts and paintings which have deteriorated over the course of time. Using pde's to achieve the task of restoration is very prevalent these days and has been evolving very frequently.

A numerical method for discretizing the Laplace equation helps us to replace the corrupted pixels in the image matrix by running numerous iterations of the formulation. Prior work done in Inpainting uses various methods. Some of the methods have briefly been discussed in this section. Diffusion principle based Inpainting is one of the pioneer approaches in this segment. Using this method, the noised region is filled by diffusing the nearby pixel values into the region to be inpainted. The diffusion based algorithms are based on Partial Differential equations (PDE). This kind of strategy shows good results when the area to be filled doesn't have a definite texture or the region is small compared to the whole image. Although, there are some problems, which include blurring of the region when the region is relatively large, these methods work well for smaller and non-textured regions. Another primitive inpainting approach is by using texture synthesis based algorithms. These algorithms fill the region by copying matching textures from the nearby pixels. These synthesis based algorithms determine the pixel values given an initial starting value, and tries to keep the texture and structure of the image nearly the same [1]. All the earlier inpainting techniques utilized these methods to fill the missing region by taking a few pixels and copying them to the neighbouring area. Markov Random Fields (MRF) are used to characterize the pixel distribution in the neighbourhood. The new texture is found by determining all the nearby texture structures. The aim of this approach is to generate textures in such a way so as to maintain a similarity to the given sample pattern and at the same time retain the statistical characteristics of the base structure [2]. Post this there was the advent of PDE based techniques. These algorithms were iterative algorithms. The main idea introduced in these techniques is to propagate the geometric and pixel information from the edge pixels into the interior of the missing region [3]. Isophote lines are used to transfer the information in the direction of least change. This strategy is expected to produce promising results if the noisy regions are relatively small. This inspired the startegy which proposed using Total Variational Inpainting model [4]. This algorithm puts to use anisotropic diffusion and the Euler-Lagrange equation. The approach performs well for relatively small areas and for noise removal operations. However, this method does not connect broken edges and cannot generate textures. The purpose of these algorithms is to preserve the structure of the inpainting area. Therefore a blurred image is produced in this process. Another major drawback in these approaches is that the larger regions in which texture filling is required are not reproduced well [5].

A relatively new method is the exemplar approach. This approach for inpainting is widely acknowledged and operative [1]. It can be generally stated in two steps. The first step involves assignment of priority and second consists of selecting the best matching patches. The similarity is found out by certain metrics. The exemplar based process synthesises the missing region using a patch constituting similar properties from a known region. Using the filling order information, the method fills in the missing region using spatial information gathered from nearby areas. This approach works well for relatively larger regions too. Another approach is the sparse representation based approach. This method has been built upon single-image super resolution. Research suggests that sparse linear combination of elements derived from an over-complete dictionary can be used to

represent patches of images. However the formulation of an exhaustive dictionary for optimal representation of wide variety of classes comprising of image patches is in itself a typical scenario [6]. Learning through such a dictionary is a tough task; hence the process is simplified by taking up random samples from the set of training images bearing similar nature. It has been established by studies that such a dictionary along with sparse representation can provide high quality image reconstructions [7]. Some of the recent approaches [8], [3] uses Partial differential equations for structural inpainting focuses on anisotropic diffusion using heat equations. We build upon the same idea in the following sections.
Laplace equations are partial differential equations of second order. It falls under the category of elliptic partial differential equations. The solutions of the Laplace equations are harmonic functions. Potential theory is the general theory for solutions of Laplace equations. Harmonic functions being the solutions for these equations very precisely explain the nature of multiple potentials like that of fluids, electricity, spring, etc. In other words, Laplace equations are simply the expression for the steady state of the heat equations. Its utility is evident by the harmonic functions it provides as its solutions which are used to model various phenomenon.

In case of Laplace equations of two independent variables, the expression could be represented as

$$\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2} = 0$$

For a region *G*, the Dirichlet problem aims at obtaining the existence of the solution where a function (say *f*) on the boundary of *G* is equal to the harmonic function φ. This means the function itself is defined on the boundaries of the domain taken.

In case of Neumann boundary condition, instead of the function φ, the first derivative of φ is equal to some function (say *f*) on the boundary of a region *G*. This in turn implies that the derivative of the function defined on the boundary of the region determines the potential.

METHODOLOGY

We use partial differential equations for the purpose of in-painting with Dirichlet boundary conditions in our proposed approach. We look for a methodology to fill the gaps in an image for its perspective reconstruction. The heat equation fits the bill well. Fourier's heat equation describes how temperature in a material changes. Given this PDE, along with the thermal conductivity of the material, and the initial temperature condition $u(t = 0)$, it is possible to calculate the temperature at any point in the material in 2 – dimension for any given time by the equation

$$u_t = c^2\Delta^2 u = c^2(u_{xx} + u_{yy})$$

Therefore heat equation could be a possible approach to this problem, wherein we iterate the process for different time intervals [3, 8]. However, a more precise and intelligent approach would be considering a scenario wherein the time tends to infinity i.e. the temperature has been distributed well. This scenario could be better modeled using the steady state heat equation or the Laplace equation which in independent of time. The Laplace equation is given as follows

$$\Delta^2 u = u_{xx} + u_{yy} = 0$$

The heat equation is also known as the diffusion equation owing to the very fact that besides modeling temperature dynamics, the equation could also present a model for describing density dynamics in a material undergoing diffusion. It can also be used to describe processes exhibiting diffusive-like behaviour, for instance the 'diffusion' of alleles in a population in population genetics. The Laplace equation thus presents a post diffusion scenario.

In case of image analysis the heat and the Laplace equation fits well. The intensity of each color in a RGB image or grayscale intensity in a b/w image could serve as the temperature or the material susceptible to diffusion for the purpose of filling up the holes in the image. Thus the color intensity of the area surrounding the hole / the portion which is to be removed / reconstructed could be used to diffuse itself into the hole. Thus, the surrounding pixels could act as the boundary values i.e. they shall enable imposition of Dirichlet boundary conditions to the Laplace equation.

Since images are represented as matrices wherein each element represents a pixel, they essentially are of a discrete nature. On the other hand partial differential equations or other similar modeling equation requires a continuous domain. Therefore in order to apply the Laplace equation to an image, we must either develop a continuous counterpart of the image or discretize the PDE model. Herein, we shall choose the later. We employ finite-difference approximation in order to discretize the Laplace equation.

For a given PDE/ ODE in domain $x \in [a, b]$, we define

$$\Delta x = (b - a)/N$$

The derivatives of 'u' leads to *N* equation for u_i *i=1, 2,3…N*

$$u_i \equiv u(i\Delta x)$$
$$x_i = i\Delta x$$
$$y_i = i\Delta y$$

Now, we shall approximate the second partial derivatives in the PDE suing the central difference scheme,

$$\frac{d^2u}{dx^2}_{i,j} \approx \frac{u_{i-1,j} - 2\, u_{i,j} + u_{i+1,j}}{(\Delta x)^2}$$

$$\frac{d^2u}{dy^2}_{i,j} \approx \frac{u_{i,j-1} - 2\, u_{i,j} + u_{i,j+1}}{(\Delta y)^2}$$

The above equations are synonymous to

$$\frac{d^2u}{dx^2}_{i,j} \approx \frac{u(x-\Delta x, y) - 2\,u(x,y) + u(x+\Delta x, y)}{(\Delta x)^2}$$

$$\frac{d^2u}{dy^2}_{i,j} \approx \frac{u(x, y-\Delta y) - 2\,u(x,y) + u(x, y+\Delta y)}{(\Delta y)^2}$$

Putting the double partial derivatives in the Laplace equation

$$\frac{u_{i-1,j} - 2\,u_{i,j} + u_{i+1,j}}{(\Delta x)^2} + \frac{u_{i,j-1} - 2\,u_{i,j} + u_{i,j+1}}{(\Delta y)^2} = 0 \quad \text{for pixel } (i,j) \qquad (12)$$

Since, the pixels are spaced apart equally in both the dimensions, we have, $\Delta x = \Delta y$.

The equation can further be rearranged as follows

$$-4\,u_{i,j} + u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} = 0 \quad \text{for pixel } (i,j) \qquad (13)$$

Thus, we arrive at the discretized form of the Laplace equation.

Now, in an image with an inconsistency we shall essentially manually/ automatically objectify a rectangular region around the inconsistency. This particular region shall serve as the hole which is supposed to be filled by the diffused surrounding pixels. We have the accurate color intensities of the surrounding pixels which serves as the Dirichlet boundary conditions.

Using the boundary conditions, we proceed with the discretized Laplace equation for processing of the subsequent pixels.

From the equation, derived so far we notice that intensity of each pixel (i, j) in the hole region is governed by the intensity of pixels on the immediate left, right, top & bottom. The process results in formation of multiple linear equations which can further be solved using Gaussian approach. The process is repeated numerous times and on each of the pixels in order to achieve satisfactory results. The threshold for stopping could be calculated analytically or on the basis of the observations. In this way the requisite region of the image shall be -in-painted.

RESULTS

In this section we present a comparative analysis of the PDE based anisotropic scheme putting to use the heat equation as described in the previous sections with the Laplace equation based approach proposed and implemented by us. The experiments are precisely discussed for four different images. For the first two images a noise was artificially introduced in localized region and the same was in-painted using the proposed approach and the heat equation based approach. The latter two images are used in their natural posture and an object so identified by us has been removed through in-painting methodology as described in the previous sections. We tabulate the comparative results in terms of their visual in-painting quality. We then tabulate the comparative computation time for the heat equation based approach and the one proposed by us. The computation for both the methodologies were undertaken on second generation Intel® Core™ i7-2620M Processor (4M Cache, up to 3.40 GHz). An upper cap of 1500s was taken in order to stall the computation in case the computations cease convergence within this time frame.

The Table I depicts that although the heat equation based approach reduced the noise it failed to converge when an object had to be completely removed. Moreover, the time taken for the current approach is much less than the heat equation based approach (Table II).

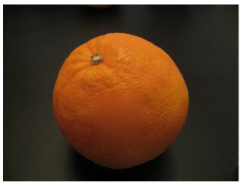
Image No.	Noisy image	In-painted image (Heat equation based approach)	In-painted image (Proposed scheme)
I			
II			
III		Failed to converge	
IV		Failed to converge	

Table I: A comparative analysis of heat equation based approach with the proposed approach

Image No.	Patch Dimensions (pixels)	Computation time for the heat equation based approach	Computation time for the proposed algorithm	Speed up
1	25 x 25	232.27	1.3	178
2	20 x 20	89.81	2.16	41
3	110 x 110	Failed to converge	1.639	-
4	100 x 50	Failed to converge	1.27	-

Table II: Description of comparative computation time for the proposed approach against the heat equation based approach

DISCUSSION

The results enumerated in the previous sections present encouraging findings. As described in Table I the heat equation based approach was either visually equivalent or less competent in terms of their in-painting quality then the proposed approach. The same could be an expected finding since the heat equation based approaches inculcate limited number of integration before a convergence criteria is satisfied, while the Laplace equation based approach inculcate infinite number of iteration since the same presents a steady state for the heat equation.

Table II in the previous section presents a comparison of the computation time for the widely used heat equation based approach and the proposed Laplace equation based approach. The latter turns out to be better in terms of the analytical time complexity and the same is reflected in the experimental results. A speedup of 178 times was observed for image I while a speedup of 41 times was observed for image II. The difference in the speedups could be attributed to system scheduling and warm time for relatively small tasks. The heat equation based approach failed to converge within a reasonable time for the other two images. The heat equation based approach moreover fails to converge in a reasonable time as soon as we start increasing the patch size of the area to be inpainted.

CONCLUSION

Our proposed approach presents a reasonable improvement in the in-painting quality. Moreover, our major contribution lies in the tremendous speedups post algorithmic improvements as proposed in our implementation. The approach enables in painting reasonably large pixel sets in a practically feasible time while the approach previously being used failed to converge even in 800-900 times the computation time taken by our proposed approach.

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